explaining the processes of business. How economy is practiced by men devoting their whole energies to the production of one thing and the manner of procuring the other necessities of life by purchasing them with the proceeds of the sale of what they produced. Then move on to discounts; showing how goods can be bought cheaper for cash than on time. From this naturally would develop the fact that business as now conducted requires a great deal of money, that it is perfectly proper for men to borrow money to pay for raw material and wages, but that the sooner they are paid for what they produce the shorter the time required to use borrowed money, and the sooner they are out of debt the smaller the expenditure for interest. This will also indicate the advantages in *giving* a cash discount and will lead up to the subject of bank loans and interest and bank discount. You will observe that I am now emphasizing *application* which to my mind is essential at every step after the early period of grinding drill. I wouldn't teach anything without showing an application, and I am certain I couldn't learn without it. Interest and discount should embrace a thorough training in compound interest and discount, for a large percentage of business transactions calls for the exercise of those calculations. As arithmetic is generally looked upon, the divisions which I have mentioned do not seem to be very comprehensive, but I feel certain that the average business man would be completely satisfied if a young man entering his employ had a thorough mastery of them. No reasonable man expects a student fresh from school to take right hold of problems peculiar to his affairs. In fact, I think most men would rather have one with mind perfectly clear of preconceived notions, so long as he could perform with facility a question in mathematics after the local system had been explained. If a boy has been so trained that he can keep his mind fixed on what it is he is endeavoring to ascertain, and shows a fair degree of dexterity in handling figures anyone who possesses the power to make his meaning clear can mould the novice into his ways without difficulty. I think it is almost useless to attempt to teach exact business methods except as to general principles, for no two systems are exactly alike. In other words, the pupil should have a "thorough knowledge of integers and frac-

tions and facility in handling them and above all the power of mind which a proper instruction in the subject is capable of giving." Teach neatness, teach accuracy, teach speed in the subject mentioned, and if you turn out your boys and girls in that condition we will do the rest.

NATIONAL CITY BANK,
NEW YORK CITY.

Whatever I have tried to do in life I have tried with all my heart to do well. Whatever I have devoted myself to, I have devoted myself to completely: in great aims and in small I have always been thoroughly in earnest.—*David Copperfield.*

EDUCATIONAL VALUES OF GEOMETRY.

By F. F. Decker.

It is doubtless quite unnecessary to point out to an audience of members of the Association of Teachers of Mathematics in the Middle States and Maryland the nature of mathematics or in particular of Euclidean geometry; yet because of its importance as a foundation for an intelligent discussion of methods of teaching geometry we shall refer briefly to it.

Mathematics is, not to attempt too great a refinement, the science of implications; and its work therefore consists in formulating postulates and tracing the inevitable conclusions. Euclidean geometry, the first geometry for the race and for the student, is characterized, in addition to its indefinables and the laws of deduction which it postulates, by its own postulates. The work of the student is to comprehend them and to make various chains of deductions. To perform this work is to study geometry.

Geometry as a science has then no connection with the phenomena about us—is not a natural science. But to assist the student to master the subject the teacher may find it necessary, yes, will many times find it necessary, to refer to the child's sense perception. In forming the concept of Euclidean space one will probably make a comparison with physical space. Thus if free mobility is to be postulated, the difficulty of moving a physical body without changing its size on account of variation of pressure may be illustrated by the case of a balloon sinking in the ocean. Again the difficulty of moving a physical body to any position on account of the impenetrability of physical matter may be pointed out. The former illustration (concerning change in size due to pressure) will suggest to the teacher that a geometry may be constructed containing the postulate that the size of the figure decreases in proportion to the distance it is moved. The successful teacher's mental storehouse will contain many such comparisons to be used when needed, but they will not be considered as geometry.

Again all teachers will have the pupil draw figures, but will

consider what seems to happen on the figure as drawn as a matter to be investigated rather than as a deduction. For example in the usual proof of the proposition that every point in a plane equi-distant from two points A and B is on the per-

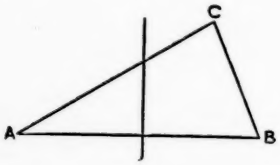

FIG. 1.

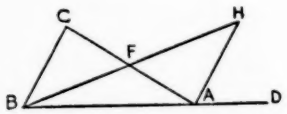

FIG. 2.

pendicular bisector of the line segement joining them by pointing out the inconsistency in assuming that such a point C may not be, one of the lines CA or CB in the figure as drawn seems to cross the perpendicular bisector of AB.

As another instance, in the proof of the theorem that an exterior angle of a triangle ABC is greater than either opposite interior angle, says B, the line segment drawn from the vertex A to the end of the line segment BH drawn to the middle of the opposite side and then produced its own length—the line segment AH—seems to be within the exterior angle so as to make the angle HAF which is the equal of ABC a part of DAC.

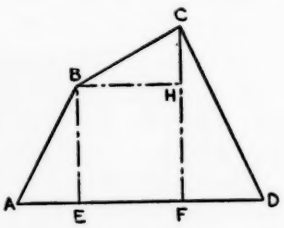

FIG. 3.

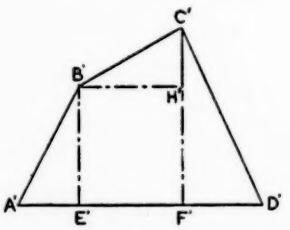

FIG. 4.

We shall give as a third instance what may appear to be a proof of the theorem that if two quadrilaterals have three sides of the one equal to three sides of another, each to each, and the two corresponding angles adjacent to the fourth sides equal, each to each, the quadrilaterals themselves are equal.

Let the given quadrilaterals $ABCD$ and $A'B'C'D'$ have $AB = A'B'$, $BC = B'C'$, $CD = C'D'$, $\angle DAB = \angle D'A'B'$ and $\angle CDA$

$= \angle C'D'A'$. Draw BE and $CF \perp$ to AD, $B'E'$ and $C'F' \perp A'D'$, $BH \perp CF$ and $B'H' \perp C'F'$.

Proof: $\triangle BAE = \triangle B'A'E'$, and $\triangle CDF = \triangle C'D'F'$ } Right $\triangle$ with hypotenuse and acute $\angle$ equal.

$\therefore BE = B'E'$, $CF = C'F'$, and $CH = C'H'$,

$\therefore \triangle BHC = \triangle B'H'C'$, $\therefore BH = B'H'$,

$\therefore BHFE = B'H'F'E'$,

$\therefore BAE + CDF + BHC + BHFE = B'A'E' + C'D'F' + B'H'C' + B'H'F'E'$, or, as the whole equals the sum of its parts,

$$ABCD = A'B'C'D'.$$

That an assumption has been made in this proof that something that seems to happen in the figure necessarily happens may be seen by a simple method of showing the falsity of the conclusion. The following construction due to Dr. R. L. Moore will show that two quadrilaterals satisfying the hypothesis may be constructed as unequal as we like.

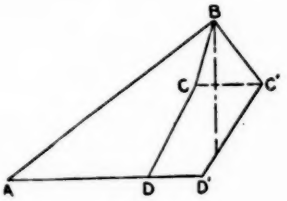

FIG. 5.

Produce a line segment AD making it any greater length, AD'. Draw angles ADC and $AD'C'$ equal. Take DC and $D'C'$ equal and take B on the perpendicular bisector of CC'. The quadrilaterals $ABCD$ and $ABC'D'$ satisfy the hypothesis without being equal.

The last instance will illustrate the fact that geometry knows no necessity except the logical and will commend to the teacher the advisability of putting the other two on logical foundations. But how? Shall we complicate the machinery by deducing the second from a postulate underlying the first or shall we have a

postulate for each. Geometry, wincing under a thrust at one of its real charms, cries out for the former; but pedagogy, unlike geometry, has other necessities than the logical, and perhaps classroom experience will dictate a compromise. In this we will agree, that the teacher who substitutes in place of a rational process the observation that certain lines seem to occupy certain positions in a drawn figure is not at that point teaching mathematics. It may be noted in passing that the assumption of Figs. 1 and 2 are neither essentially true nor essentially false, for falsity is a matter of inconsistence and until a fundamental such as Pasch's assumption is introduced, there is nothing with which they can be inconsistent.

True it is that the question is often raised if real geometry can actually be taught to the student in the high school. A society of mathematics teachers like ours, with a high regard for the real beauty of the science, with a keen appreciation of its cultural aspect, will hesitate to deny it. No teacher will answer in the negative until he has familiarized himself with a geometry foundation, consistent but irreducible, such as that of Hilbert or Veblen. Hilbert himself says that geometry made more nearly rational is thereby made more simple. Halsted—and it seems a pity that the name of a man with so ardent a love for pure geometry should not have appeared in the bibliography of science teaching published during the current year for the American Federation of Teachers of the Mathematical and the Natural Sciences, even if only to make intelligible to the reader the answer made to him by Professor Dewey—Halsted seems to believe that rational geometry can be taught to the high school student. We will perhaps hold such a logical system as purely an ideal toward which we will endeavor to make steady advance. Such men cannot be ignored and if we are at any point to teach something else for geometry these men must be answered. The really important consideration seems to be that we make the effort to approach the standard set by the geometer.

And why? Because, for one reason, the opportunity to teach geometry at all is at stake. But why should geometry be taught at all? We may answer: because of the beautiful deductive system it puts before the student; because of the stimulus this system furnishes to the systematizing of bodies of knowledge in

other fields. We must be in a position to answer: because of the power the student acquires in framing hypotheses, in sizing up situations, and in deducing conclusions. These things the student must be asked to do throughout the course. These things he must do in life; or rather we would say, the potentiality for training, to a greater or less degree in these directions forms a material part of his birthright.

Says Clifford: "This book (Euclid) has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state. The encouragement; for it contained a body of knowledge that was really known and could be relied on. The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained."

A list of concomitant benefits of the study of geometry of considerable length might be given, including such as skill in mechanical drawing, development of the space intuition, of accuracy in computation, and the application to physical space; but we are considering those benefits that have their source in the nature of the subject itself and we mention only these: the appreciation of a type of the beautiful and the useful, the stimulus to follow the model and the ability so to do.

Geometry teaching suffers from the fact that the subject seems to have a practical application. When a teacher becomes weary of the great task of assisting pupils through this system of deductive reasoning he is exposed to the danger of forsaking his real work for that of some practical application. He will say, perhaps, that his pupil is going to enter some trade and is in most need of specialized manual skill. What a pity! For if one student more than another needs to be able to think it is the one whose environment is in the greatest need of improvement.

Not long since the writer was privileged to hear a discussion of the advisability of establishing a vocational high school, led by a manufacturer and a journeyman tradesman. The manufacturer dwelt on the necessity of every man earning a living, of the necessity of getting recruits for the trades, and the advantage resulting from the co-operation between the factory

(*To be continued.*)

NEW BOOKS.

Differential and Integral Calculus. By LORRAIN S. HULBURT. New York: Longmans, Green, and Company. Pp. 481.

This book is intended for use in colleges and technical schools and is rather carefully written. It contains many excellent features which will appeal to the teacher and is one of the best books for the purpose that has come to our attention. It would seem, however, that too many writers on the calculus have aimed to meet the requirements of these two classes of students and in so doing have not entirely satisfied the desires of either. The teacher of the arts students will want to omit a good deal that the engineering students will want and *vice versa*. While the theory of the calculus is the same for all we believe it would be more satisfactory under present conditions to have a separate book for each class of students.

Elements of Trigonometry with Tables. By DANIEL A. MURRAY. New York: Longmans, Green, and Company. Pp. 231. $1.00 net.

In some respects this book is an abridgment of the author's "Elements of Trigonometry" with some rearrangement of chapters. For most teachers we believe it will be found an improvement on the earlier book, though the increased use of the unit circle and line functions is not desirable. Some teachers, however, may desire this increased use of line functions and for them it will be an improvement. It would appear to be a very teachable book for those who desire to begin with a treatment of acute angles.

Elements of Applied Mathematics. By HERBERT E. COBB. Boston: Ginn & Company. Pp. 274. $1.00.

This book is constructed on the principle that arithmetic, algebra, and geometry are closely linked together. The work outlined consists largely of lists of problems based on the student's preceding work in mathematics, illustrating the work in the shops and laboratories, and of simple experiments and exercises in the mathematics classroom, where the pupil, by measuring and weighing, secures his own data for numerical computations and geometrical constructions.

The College Engineering Notebook. By ROBERT E. MORITZ. Boston: Ginn & Company. $1.00.

This notebook is designed for the use of students in civil, mechanical, and electrical engineering. It contains ninety sheets of high-grade rectangular-co-ordinate paper, five sheets of polar-co-ordinate paper, and five sheets of logarithmic-co-ordinate paper. The reverse side of each sheet is ruled horizontally with vertical crosslines at intervals of two and one half centimeters. This arrangement adapts each sheet

equally well for computation purposes, for the construction of graphs, and for ordinary use.

Additional features of the book are lists of the most important mathematical formulas from algebra, geometry, trigonometry, analytics, and the calculus. These are followed by separate pages of carefully selected formulas from mechanics, surveying, strength of materials, mechanism, machine design, thermodynamics, electricity, and magnetism. The back part of the book contains a four-place table of logarithms, and short tables of natural logarithms, trigonometric functions, exponential functions, squares and square roots, cubes and cube roots, reciprocals, and hyperbolic functions. Finally, it contains eight sets of type curves.

Complete School Algebra. By H. E. HAWKES, W. A. LUBY, F. C. TOUTON. Boston: Ginn and Company. Pp. 507. $1.25.

The "Complete School Algebra," which includes between the covers of a single volume—with the necessary adaptation and abridgment—all the material of the authors' "First Course in Algebra" and "Second Course in Algebra," is designed for those schools which find a one-book course best suited to their needs.

The first twenty-three chapters contain the greater portion of the work usually taken up during the first year. Then follows the review material, each topic being given a broader and more advanced treatment than is permissible in first-year work. New matter is used throughout, and many new applications are given in order to make a fresh and inviting appeal to the student. In the remaining chapters those advanced topics considered necessary by the best secondary schools are included.

First Principles of Algebra. By H. E. SLAUGHT and N. J. LENNES. Boston: Allyn and Bacon. Pp. 280.

The authors in writing this book have kept two aims before them: (1) To provide a gradual and natural introduction to the symbols and processes of algebra. (2) To give vital purpose to the study of algebra by using it to do interesting and valuable things. The equation is introduced and developed early, and the principles are codified in a few short rules. It is an interesting and carefully written book.

First Year Algebra. By WEBSTER WELLS and WALTER W. HART. Boston: D. C. Heath and Company. Pp. 340. 90 cents.

This book shows some new blood has been infused into the older editions of Wells's book. It contains several excellent features and a student who had carefully completed it would have a good introduction to algebra.

Durell's School Algebra. By FLETCHER DURELL. New York: Charles E. Merill Co. Pp. 507. $1.10.

This is an attractive and interesting book. Its pages are well spaced and give an effect of simplicity to the text.

The introduction of formal algebra is gradual and the order seems teachable. One of the best features from the standpoint of pedagogy is the handling of exponents and radicals, their relation being used to advantage.

There are, as is usually the case, some features to which exception might be taken; for example, the failure to classify the cases of factoring under as few heads as possible, the emphasis given the vx method of solving simultaneous equations, rather than to the method by elimination of the independent term, etc.

Among other features worthy of mention are the historical material, the sets of review examples, the placing in the appendix of topics of doubtful value in an elementary course, and the introduction of material for problems, the pupils being required to make their own questions.

Plane Trigonometry and Tables. By FLETCHER DURELL. New York: Charles E. Merrill Co. Pp. 298. $1.25.

This is an excellent trigonometry, both in subject matter and in arrangement. One of its strong features is the way in which the subject is made to appear reasonable by a clear exposition of the advantages of certain of its methods and formulas.

The tables are complete, containing tables for decimal parts of a degree as well as for minutes, and they are easy to the eye.

The methods used are, as a whole, modern and clearly set forth. The arrangement of logarithmic work is good, and the method of indicating the given and required parts of triangles is simple and effective.

In so strong a book, it seems a pity that the author should have overlooked so simple a generalization as should be used in the proof of the Law of Lines. It seems as if authors of trigonometry fix their minds so firmly on function of acute angles that they overlook the possibilities of angles of more than ninety degrees.

The Francis W. Parker School Year Book. Vol. I. By the FACULTY OF THE FRANCIS W. PARKER SCHOOL. Chicago. Pp. 140. Price 35c. Press of the Francis W. Parker School, 330 Webster Avenue, Chicago, 1912.

The faculty of the Francis W. Parker School, Chicago, has undertaken to publish, annually, a volume, entitled *The Francis W. Parker School Year Book*, consisting of a series of concrete illustrated reports of work actually done in the school.

This school is in the nature of an educational laboratory, or an experiment station in education. Being unhampered by the traditions that beset the average school, and free to experiment in any way with the content of the course of study and methods of organization and teaching, the school is endeavoring to carry out certain modern fundamental principles and aims in education—principles and aims advanced especially by Colonel Francis W. Parker, one of the pioneers in the modern educational movements in this country.

The school proposes to share, with the general educational public, the benefits of its experimentation, through the Year Book. Each volume will be devoted to some one phase of education as actually worked out in the school.

Volume I. is devoted to the *social motive in school work*, and consists of a number of reports dealing with phases of handwork, music, dramatics, etc., in which the *social motive* predominates. The reports are plentifully illustrated—thirty-six illustrations in all—and show in a concrete way how the projects or activities described are made use of in the class room. The experiences of the school are thus suggested in dealing with the great educational problems of motivation, correlation, etc., of the present day. The volume should be of immense practical value to every teacher in the country.

Work and Play With Numbers. By GEORGE WENTWORTH and DAVID EUGENE SMITH. Boston: Ginn and Company. Pp. 144.

NOTES AND NEWS.

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